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PHY xxx: Advanced Quantum Mechanics

Johannes Kirscher

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E-mail: kirscjo@gmail.com
Office Hours: Tu(10:45 am) Class Hours: M(9:45 am), Th(9:45 am), F(10:40 am)
Office: ? Class Room: virtual (tba $(t_{\text{class}} - t) > 1$ h)

Lab Room: ?? Lab Hours: Tu(3:35, 4:30)

When Schrödinger first wrote it [his famous equation] down, he gave a kind of derivation based on some heuristic arguments and some brilliant intuitive guesses. Some of the arguments he used were even false, but **that does not matter: the only important thing is that the ultimate equation gives a correct description of nature.**

(R .P .Feynman)

1 particle identity and statistics

For a detailed exposition see volume 1, appendix 1C in [3], and [1] for Bose's original usage of particle statistics of indistinguishable particles to derive the Planck distribution.

1.1 Bose and Fermi statistics

Define the notation:

$$|\alpha\rangle \otimes |\beta\rangle \equiv |\alpha\beta\rangle \qquad \text{particle one in state } |\alpha\rangle\text{, particle two in state } |\beta\rangle \quad ,$$
$$|\beta\rangle \otimes |\alpha\rangle \equiv |\beta\alpha\rangle \qquad \text{particle one in state } |\beta\rangle\text{, particle two in state } |\alpha\rangle \quad ,$$

but since either state would allow us to label particles with the state's quantum numbers, which would defy the notion of indistinguishability, physical states must allow for some probability to find a particle in any state, while this probability obeys the

Symmetrization postulate:

- The states of particles whose spin is an integer multiple of $\hbar$ are symmetric and are called bosons.
- States of particles whose spin is a half odd-integer multiple of $\hbar$ are anti-symmetric and are called fermions.
- Partially symmetric states do not exist but, nevertheless, are referred to as paraparticles.

1.1.1 Pauli principle

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Wick's theorem:

An ordinary product of any finite number of creation and annihilation operators is equal to the sum of normal products from which $0, 1, 2, \ldots$ contractions have been removed in all possible ways.

First, make sure you do understand all terms which appear in this theorem and formulate your own example. Does it matter which kind of particle state the operators create/annihilate? Check your example operator string for Fermi and Bose particles.

Gaussian-integral formula (ch. 3.2 in [4]):

The Gaussian integral over the N-components v_i of the vector $\mathbf{v}$ parametrized by the positive-definite, real, symmetric matrix $\mathbf{A}$

$$(2\pi)^{-N/2}\sqrt{\det(\mathbf{A})}\int\limits_{\mathbb{R}^N} \mathrm{d}\mathbf{v}\; e^{-\frac{1}{2}\mathbf{v}^\dagger \mathbf{A}\mathbf{v}}\,(\ldots) \equiv \langle \ldots \rangle$$

evaluates for an arbitrary product of components

$$\langle v_{i_1} v_{i_2} \ldots v_{i_{2n}} \rangle = \sum_{\substack{\text{pairings of}\\ \{i_1, i_2, \ldots, i_{2n}\}}} \left(\mathbf{A}^{-1}\right)_{i_{k_1} i_{k_2}} \ldots \left(\mathbf{A}^{-1}\right)_{i_{k_{2n-1}} i_{k_{2n}}} \tag{1}$$

Again, do not proceed without having realized the obvious:

- What is the integral's value for a product string of an odd number of components?
- Can the overlap be non-zero if any of the components appears in the string not with an even power?

Now, Wick's theorem facilitates the calculation of matrix elements between physical states $|n_1 n_2 \ldots n_k\rangle$ for a string of creation and annihilation operators, e.g., for the expectation value of an interaction Hamiltonian

$$V = \frac{1}{4}\sum_{\alpha,\beta,\gamma,\delta} \langle \alpha\beta|v|\gamma\delta\rangle\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta \quad .$$

Express the matrix $\mathbf{A}$ and the vectors $\mathbf{v}^{(\dagger)}$ in terms of the creation and annihilation operators $c_\alpha^{(\dagger)}$ and the associated states $|\alpha\rangle$. Then, generalize (1) such that it also applies to Fermi operators.

1.2 Constructing many-body wave functions with the right symmetries from single-particle states

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Python/Mathematica script which lists all *Young tableaux* and *tables* for a generic partition.

2 Approximations

2.1 Monte-Carlo sampling

2.2 variational technique

2.3 perturbation theory

2.4 Hartree-Fock approximation [2]

3 Scattering theory

Scattering experiment:

A projectile beam is prepared in the infinite past from where it enters a finite region of space-time in which it interacts with a target over a finite time interval before being detected in the infinite future when the interaction with that target is, again, null.

3.1 The cross-section

4 Radiation

4.1 V-A interaction exemplified the photo-dissociation of the deuteron

References

[1] Bose, “Plancks Gesetz und Lichtquantenhypothese,” *Zeitschrift fur Physik*, vol. 26, pp. 178–181, Dec. 1924.

[2] D. R. Hartree, “The wave mechanics of an atom with a non-coulomb central field. part i. theory and methods,” *Mathematical Proceedings of the Cambridge Philosophical Society*, vol. 24, no. 1, p. 89110, 1928.

[3] A. Bohr and B. R. Mottelson, *Nuclear Structure*. World Scientific Publishing Company, 1998.

[4] A. Altland and B. D. Simons, *Condensed Matter Field Theory*. Cambridge University Press, 2 ed., 2010.